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Spinning Projectile Instability Induced by an Internal Mass Mounted on an Elastic Beam

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The steady-state motion of a loose internal component can cause flight instabilities. In previous work, these instabilities have been predicted in terms of the amplitude and phase of the component motion. In this report, for the case of a component mounted on an elastic beam, the amplitude and phase of the motion are predicted in terms of the beam characteristics and the component's mass properties. Thus, flight stability can be computed from design parameters.

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1. INTRODUCTION

In the mid 1950s four shell types exhibited unusual behavior that involved the movement of their interior parts (Boyer 1955a, 1955b; Roecker and Boyer 1957; Karpov and Bradley 1958). In 1977 a simple theory was developed to relate the motion of internal parts to the angular motion of the spinning projectile (Murphy 1977). This theory assumed a circular lateral motion of the internal component combined with a coning motion of its axis of symmetry. Both of these motions are at a coning frequency of the spinning shell. The frequency of the internal motion was assumed to be the fast frequency of the projectile's free angular motion, the amplitude was set by the available tolerances, and the phase of the motion was induced by internal friction.

The theory predicted a destabilizing side moment and a spin-up moment equal to the product of the side moment and the magnitude of the projectile's angular motion. Later it was shown that this relation between side moment and spin moment is valid for any steady motion of an internal payload (Murphy 1989). For 60° phase angles, all observed instabilities could be explained by the measured clearances and the measured despins of the unstable projectiles were in good quantitative agreement with the theory's prediction.

The theory, however, made no attempt to predict the amplitude and phase of the internal motion. For a mass supported by an elastic internal beam, W. R. Chadwick (1975) did develop a complete theory for the combined motion of the internal mass and the projectile. Unfortunately, this work was marred by two errors in the proper treatment of beam damping, and his equations did not predict flight results.

In this report,* we develop the correct equations of motion for the projectile and its internal, beam-supported mass. Using Chadwick's influence coefficients, we compute the elastic response of various beams with a proper expression for beam damping. The beams considered are forward-facing and rearward-facing cantilever beams, fixed-pinned beams, and fixed-fixed beams.

The theory and associated computer runs show that the instabilities induced by beam-mounted masses are strongly controlled by beam elastic characteristics and, very importantly, the form and amplitude of

* A shorter version of this report has appeared as an AIAA preprint (Murphy 1992).

the beam damping. Small beam damping has little effect on projectile flight stability, but moderate damping can cause instability when the beam is sufficiently soft.

The inertia properties of the internal mass determine the relative importance of beam deflection or beam cant on projectile stability. Theory indicates design tradeoffs for beam characteristics and mass inertial properties.

2. EQUATIONS OF MOTION

In Murphy (1977), the internal component is assumed to perform a known lateral and angular motion. In complex notation, the lateral motion is described in nonspinning coordinates by

$$y_c + iz_c = \ell E, \tag{1}$$

and the angular motion is described by the component's axis of symmetry orientation angles with respect to the projectile's axis of symmetry:

$$\gamma_y + i\gamma_z = \Gamma. \tag{2}$$

The projectile's motion relative to its flight path is given by its angles of attack and sideslip in nonrolling coordinates:

$$\tilde{\beta} + i\tilde{\alpha} = \tilde{\xi}. \tag{3}$$

These geometrical quantities are shown in Figures 1 and 2. The usual linear analysis yields the following equation for the projectile's motion:

$$I_t \ddot{\tilde{\xi}} - (A_q + iL_x)\dot{\tilde{\xi}} - (A_\alpha + iA_{p\alpha})\tilde{\xi} = B_\varepsilon \ddot{E} + I_{tc}\ddot{\Gamma} - i p_c I_{xc} \dot{\Gamma}. \tag{4}$$

The angular motion of a solid projectile ($E = \Gamma = 0$) is usually described as the sum of two coning motions of the form

$$\tilde{\xi} = \sum_{j=1,2} K_j e^{i\phi_j}, \tag{5}$$

$$\dot{\phi}_j = \frac{L_x}{2I_t}\left[1 \pm \sqrt{1 - 1/s_g}\right], \tag{6}$$

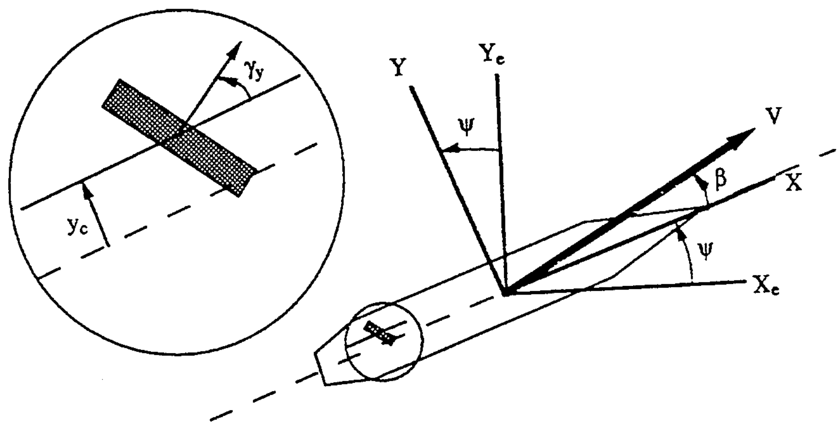

Figure 1. X-Y coordinates for projectile and internal component; $x_c < 0$.

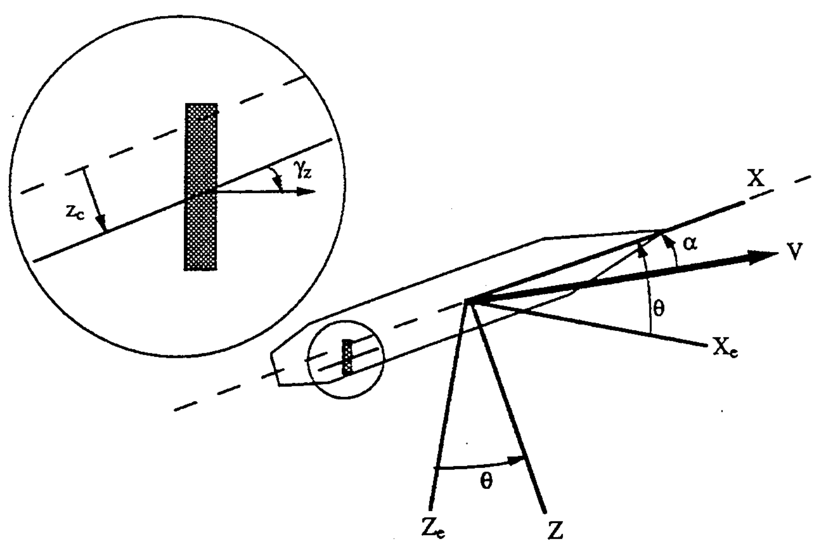

Figure 2. X-Z coordinates for projectile and internal component; $x_c < 0$.

$$s_g = \frac{L_x^2}{4 I_t A_\alpha}, \tag{7}$$

and

$$\frac{\dot{K}_j}{K_j} = \lambda_j = \frac{\dot{\phi}_j A_q + A_{p\alpha}}{2\dot{\phi}_j I_t - L_x}. \tag{8}$$

Since the fast mode is usually the one adversely affected by payload motion, a simple one-mode motion is assumed, and the internal component is assumed to perform constant-amplitude motion at the fast frequency.

$$\Gamma = \gamma e^{i(\phi_1 + \phi_\gamma)} \tag{9}$$

and

$$E = \varepsilon e^{i(\phi_1 + \phi_\varepsilon)}. \tag{10}$$

When equations 9–10 are substituted in equation 4, it can be shown that the frequency and damping of the fast mode are changed by the following amounts:

$$\Delta \dot{\phi}_1 = \frac{-\dot{\phi}_1 C_1}{K_1 (2 I_t \dot{\phi}_1 - L_x)} \tag{11}$$

and

$$\Delta \lambda_1 = \frac{\dot{\phi}_1 S_1}{K_1 (2 I_t \dot{\phi}_1 - L_x)}, \tag{12}$$

where

$$C_1 = B_\gamma \gamma \cos \phi_\gamma - B_\varepsilon \dot{\phi}_1 \varepsilon \cos \phi_\varepsilon$$

$$S_1 = B_\gamma \gamma \sin \phi_\gamma - B_\varepsilon \dot{\phi}_1 \varepsilon \sin \phi_\varepsilon$$

$$B_\gamma = I_{xc} p_c - I_{tc} \dot{\phi}_1$$

$$B_\varepsilon = m_c x_c \ell.$$

γ and ε and their phase angles were estimated from available clearances and good engineering predictions were obtained. In this report, we will predict these quantities for a component mounted on an elastic beam.

3. COMPONENT EQUATIONS OF MOTION

The lateral motion of an internal component can be described in a nonrolling coordinate system whose x-axis is aligned along the projectile's axis of symmetry. The angular velocity of these coordinates can be specified by

$$Q = \dot{\theta} + i\,\dot{\psi}.$$

The lateral motion of the component can be easily expressed in terms of the transverse components of the force and moment exerted by the beam on the component:*

$$m_c \left[\ell \ddot{E} + V \left(\dot{\tilde{\xi}} - i\,Q \right) - i\,x_c \dot{Q} \right] = F_{yc} + i\,F_{zc} \tag{13}$$

and

$$I_{tc} \left(\ddot{\Gamma} - i\,\dot{Q} \right) - i\,p_c I_{xc} \left(\dot{\Gamma} - i\,Q \right) = -\left(M_{yc} + i\,M_{zc} \right). \tag{14}$$

The elastic part of the beam force and moment can be computed by simple beam theory to be a linear function of E and Γ. For the coning motion of equations 9–10,

$$\left[F_{yc} + i\,F_{zc} \right]_{elastic} = \left[b_{11}\,\ell\varepsilon\, e^{i\phi_\varepsilon} + b_{12}\,\gamma e^{i\phi_\gamma} \right] e^{i\phi_1} \tag{15}$$

and

$$\left[M_{yc} + i\,M_{zc} \right]_{elastic} = i\left[b_{21}\,\ell\varepsilon\, e^{i\phi_\varepsilon} + b_{22}\,\gamma e^{i\phi_\gamma} \right] e^{i\phi_1}, \tag{16}$$

* See Appendix A for derivation of Equations 13–14. Appendix B outlines the derivation of Table 1.

where $b_{12} = b_{21}$ and the linear coefficients, b_{ij}, can be computed for a variety of beam supports by taking the matrix inverse of the influence coefficients, a_{ij}, given in Table 1.

The effect of beam damping can be approximated by linear terms in the derivatives of E and Γ. Since the beam is spinning with the projectile, these derivatives must be taken in projectile-fixed coordinates

Table 1. Influence Coefficients for Various Beams

Type	$a_{11}\hat{E}I$	$a_{21}\hat{E}I$	$a_{22}\hat{E}I$
Fixed-fixed	$\frac{a^3b^3}{3(a+b)^3}$	$-\frac{a^2b^2(a-b)}{2(a+b)^3}$	$\frac{ab(a^2+b^2-ab)}{(a+b)^3}$
Fixed-pinned	$\frac{a^3b^2(3a+4b)}{12(a+b)^3}$	$\frac{a^2b(2b^2-a^2)}{4(a+b)^3}$	$\frac{a(a^3+4b^3)}{4(a+b)^3}$
Pinned-fixed	$\frac{a^2b^3(4a+3b)}{12(a+b)^3}$	$\frac{ab^2(b^2-2a^2)}{4(a+b)^3}$	$\frac{b(4a^3+b^3)}{4(a+b)^3}$
Fixed-free	$\frac{a^3}{3}$	$\frac{a^2}{2}$	a
Free-fixed	$\frac{b^3}{3}$	$-\frac{b^2}{2}$	b
Pinned-pinned	$\frac{a^2b^2}{3(a+b)}$	$\frac{ab(b-a)}{3(a+b)}$	$\frac{a^2+b^2-ab}{3(a+b)}$

that are spinning with respect to earth-fixed axes. A coning frequency $\dot{\phi}_1$ in the nonspinning system would appear in a spinning system as $\dot{\phi}_1 - p$ where p is the projectile spin. The beam damping can be inserted in equation (15) by replacing b_{11} by $b_{11}\left[1 + i\,d_\varepsilon(\dot{\phi}_1 - p)p^{-1}\right]$ and in equation (16) by replacing b_{22} by $b_{22}\left[1 + i\,d_\gamma(\dot{\phi}_1 - p)p^{-1}\right]$.

If the small effect of the lift force is neglected,

$$\dot{\tilde{\xi}} = i\,Q. \qquad (17)$$

Equations 13–16 can be combined to yield

$$\left[\dot{\phi}_1^2 m_c + b_{11}\left[1 + id_\varepsilon(\dot{\phi}_1 - p)p^{-1}\right]\right]\ell\varepsilon e^{i\phi_\varepsilon} + b_{12}\gamma e^{i\phi_\gamma} = m_c x_c(\dot{\phi}_1)^2 K_1 \tag{18}$$

and

$$b_{21}\ell\varepsilon e^{i\phi_\varepsilon} + \left[-\dot{\phi}_1 B_\gamma + b_{22}\left[1 + id_\gamma(\dot{\phi}_1 - p)p^{-1}\right]\right]\gamma e^{i\phi_\gamma}$$
$$= -i\dot{\phi}_1 B_\gamma K_1. \tag{19}$$

When beam parameters, component inertia properties, and coning motion parameters are inserted in equations 18–19, amplitude and phase angles of the component motion can be determined. From these, the stability of the coning motion follows from equation 12.

4. DISCUSSION

To illustrate these results, we will consider an 8-in projectile with a particular rotationally symmetric internal component. The appropriate physical and aerodynamic properties of this projectile and its component are given in Table 2. The nominal value of the beam's stiffness is given as $\hat{E}I* =$ 59,800 lb/ft^2. In Figures 3 and 4, the projectile damping is plotted as a function of beam elasticity for all six possible beam types. When the beam stiffness is reduced to 10% of the Table 2 value, all beam types can cause flight instability.

Table 2. Test Case Values

a	= 0.50 ft	d	= $d_\varepsilon = d_\alpha = 0.5$
b	= 1.16 ft	$\hat{E}I*$	= 59,800 lb/ft^2
I_{xb}	= 0.240 slug/ft^2	I_{xc}	= 0.130 slug-ft^2
I_{tb}	= 2.306 slug/ft^2	I_{tc}	= 0.700 slug-ft^2
m_b	= 20 slug	m_c	= 2.6 slug
ℓ	= 8 in	x_c	= –0.3 ft
C_{m_α}	= 5	λ_{10}	= –0.33 1/s
C_{n_α}	= 1.8	p	= 637 rad/s
ρ	= 0.002 slug/ft^3	V	= 1,200 ft/s

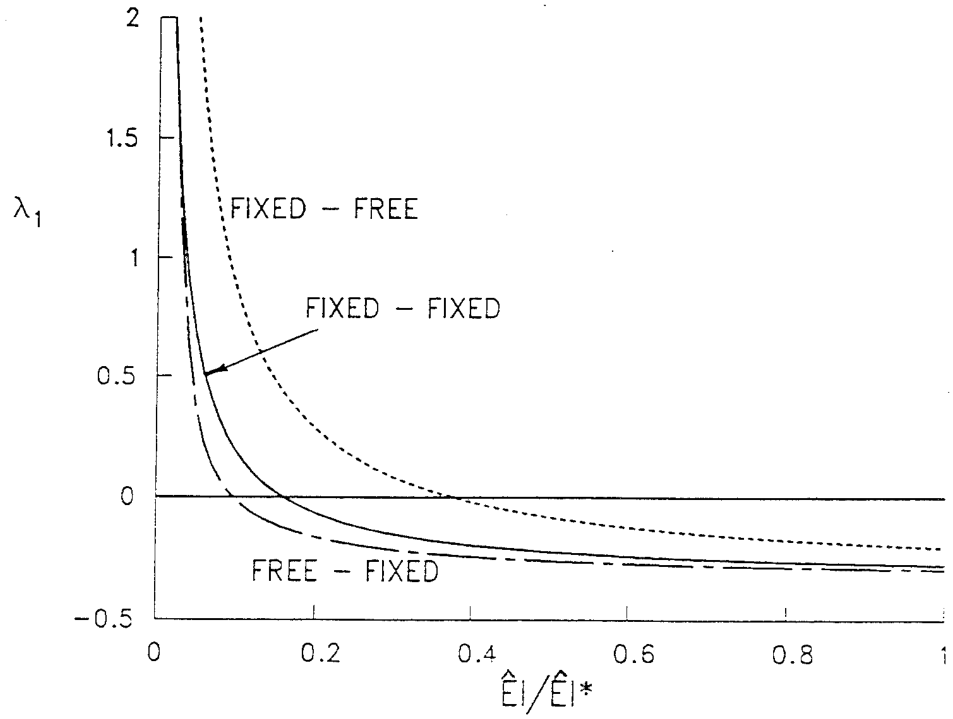

Figure 3. Projectile damping as a function of beam stiffness for three nonpinned beam types.

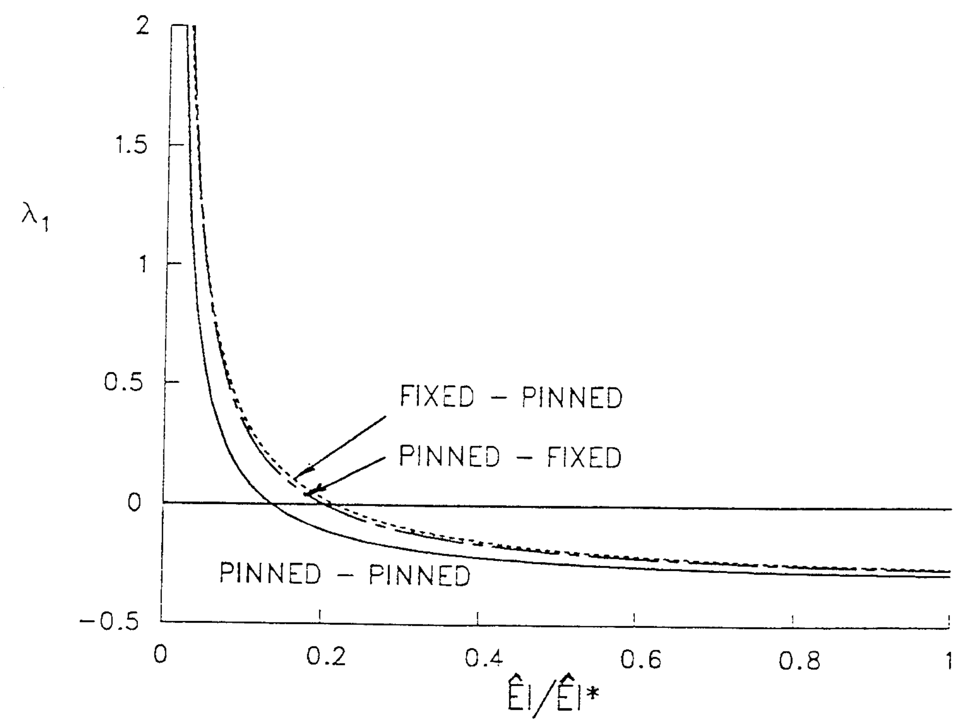

Figure 4. Projectile damping as a function of beam stiffness for three pinned beam types.

In Figure 5, the size of the component motion is plotted as a function of beam stiffness for the fixed-fixed beam. For small coning motion ($K_1 < 0.1$), this motion is quite small and only the relatively large value assigned to I_{t_c} causes this motion to induce flight instability.

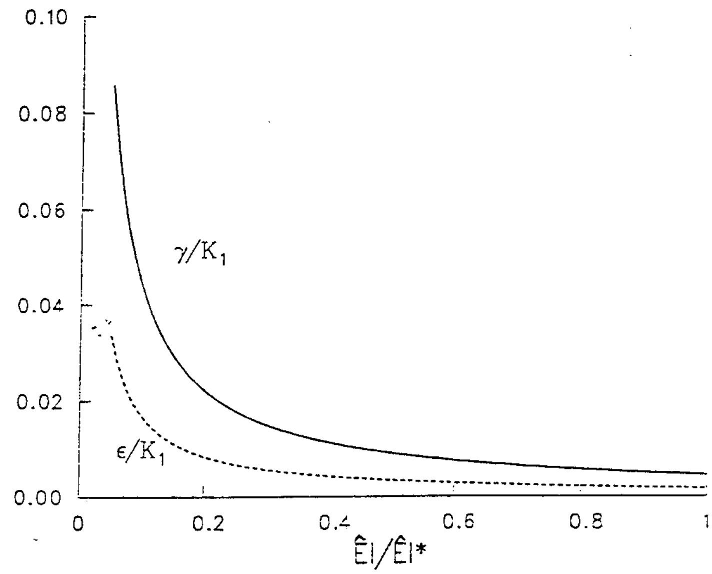

Figure 5. Component cant and deflection as functions of beam stiffness for a fixed-fixed beam.

Figure 6 shows the beam damping effect on flight stability for the fixed-fixed beam with $\hat{E}I/\hat{E}I* = 0.1$. The nominal value of $d = 0.5$ is shown for the equal damping curve $d_\varepsilon = d_\gamma = d$. The other two curves consider the solo effects of continued damping (d_γ) and of deflection damping (d_ε), respectively. Clearly, cant damping can have a greater adverse effect, and 0.5 is the worst value for $d_\varepsilon = d_\gamma$

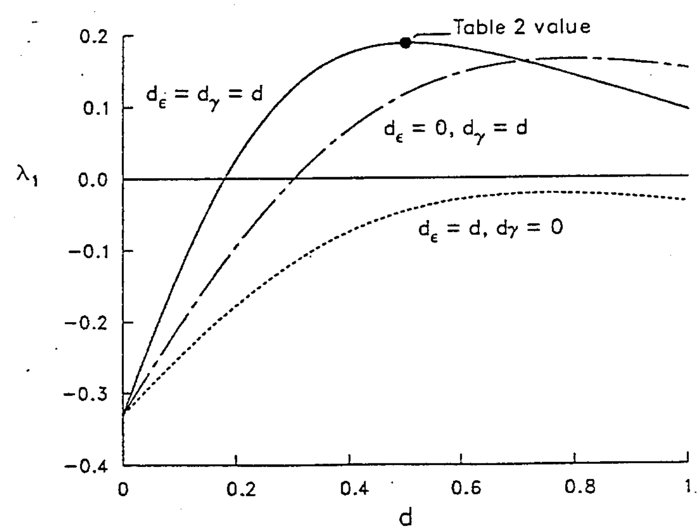

Figure 6. Projectile damping as a function of beam damping for a fixed-fixed beam and three types of damping; $\hat{E}I/\hat{E}I* = 0.1$.

Intuitively, one might think the instabilities shown by Figures 3 and 4 are due to reducing the natural frequency of the beam to the coning frequency of the projectile. In Figure 7, the natural beam frequency, ω_ε, of the three beam types of Figure 3 is computed, and the projectile damping is plotted as a function of $\dot{\phi}_1/\omega_\varepsilon$. For the fixed-fixed beam, instability occurs when the coning frequency is only 10% of the beam frequency!

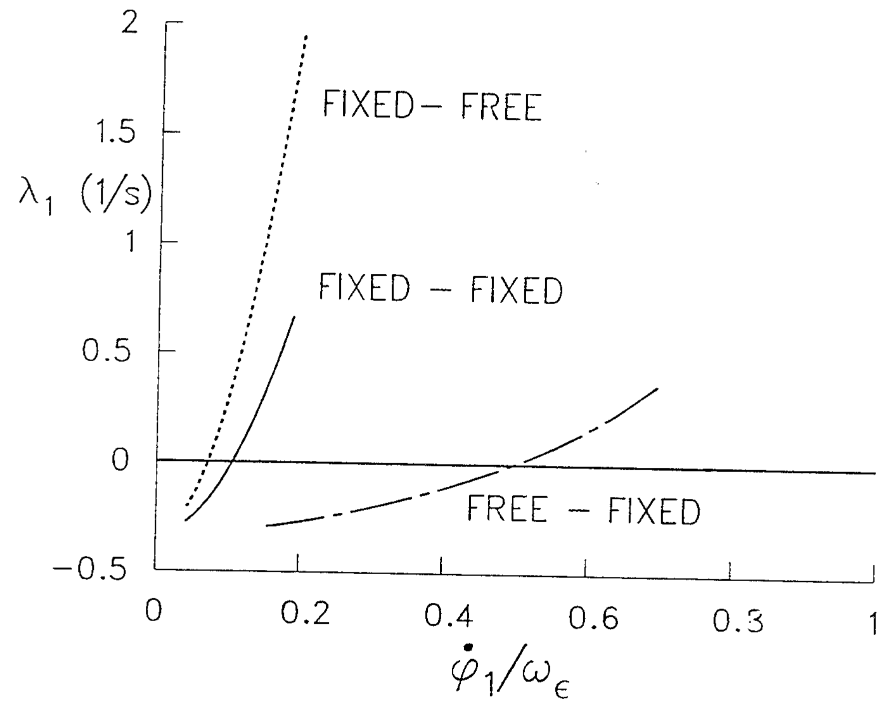

Figure 7. Projectile damping as function of coning frequency for three nonpinned beam types.

5. CONCLUSIONS

(1) The steady-state motion of internal mass on an elastic beam can be computed.

(2) For appropriate values of beam damping, flight instabilities can be predicted.

(3) These instabilities can occur for frequencies less than 10% of the beam natural frequency.

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APPENDIX A:

EQUATIONS OF MOTION

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The complete projectile is assumed to consist of a body with mass m_b and an internal component of mass m_c mounted on an elastic massless beam. The total mass of the projectile is $m = m_c + m_b$. The center of mass (CM) of the internal component is located a distance x_c from the CM of the projectile, and the CM of the body is a distance x_b from the CM of the projectile ($m_b x_b + m_c x_c = 0$). If I_{tc} is the transverse moment of inertia of the component relative to its CM and I_{tb} is the transverse moment of inertia of the body relative to its CM, then I_t, the transverse moment of inertia of the projectile relative to its CM, is given by

$$I_t = I_{tb} + m_b x_b^2 + I_{tc} + m_c x_c^2. \qquad \text{(A-1)}$$

The projectile's axis of symmetry can be located relative to fixed axes by its pitch and yaw angles, θ and Ψ, and its orientation relative to the velocity vector can be defined by its angles of attack and side-slip,* $\tilde{\alpha}$ and $\tilde{\beta}$. The internal component is assumed to be a cylinder or disk whose axis is located relative to the projectile's axis by the angles γ_y and γ_z. The transverse displacement of the component CM relative to the body's axis of symmetry is specified by the displacements y_c and z_c. All these geometrical quantities are shown in Figures 1 and 2.

The projectile has lateral aerodynamic forces and moments acting on it, represented by F_y, F_z and M_y, M_z, respectively. The elastic beam exerts forces and moments on the internal component when it is deflected; these are represented by F_{yc}, F_{zc}, and M_{yc}, M_{zc}, respectively. For simplicity, aerodynamic drag is neglected and all angles are assumed to be small.

The equations of motion of the projectile body and internal component are as follows:

$$m_b \left[V\,(\dot{\tilde{\beta}} + \dot{\psi}) + x_b \ddot{\psi} \right] = F_y - F_{yc}, \qquad \text{(A-2)}$$

* The tilde superscripts are used to indicate that these angles are measured in nonspinning coordinates and not the usual missile-fixed coordinates.

$$m_b \left[V \left(\dot{\tilde{\alpha}} - \dot{\theta} \right) - x_b \ddot{\theta} \right] = F_z - F_{zc} , \tag{A-3}$$

$$I_{tb} \ddot{\theta} + p_b I_{xb} \dot{\psi} = M_y - M_{yc} + x_b F_z + \left(x_c - x_b \right) F_{zc} , \tag{A-4}$$

$$I_{tb} \ddot{\psi} - p_b I_{xb} \dot{\theta} = M_z - M_{zc} - x_b F_y - \left(x_c - x_b \right) F_{yc} , \tag{A-5}$$

$$m_c \left[\ddot{y}_c + V \left(\dot{\beta} + \dot{\psi} \right) + x_c \ddot{\psi} \right] = F_{yc} , \tag{A-6}$$

$$m_c \left[\ddot{z}_c + V \left(\dot{\alpha} - \dot{\theta} \right) - x_c \ddot{\theta} \right] = F_{zc} , \tag{A-7}$$

$$I_{tc} \left(\ddot{\theta} - \ddot{\gamma}_z \right) + p_c I_{xc} \left(\dot{\psi} + \dot{\gamma}_y \right) = M_{yc} , \tag{A-8}$$

and

$$I_{tc} \left(\ddot{\psi} + \ddot{\gamma}_y \right) - p_c I_{xc} \left(\dot{\theta} - \dot{\gamma}_z \right) = M_{zc} , \tag{A-9}$$

where I_{xb} and I_{xc} are the axial moments of inertia of the body and component, respectively.

The transverse force on the component can be eliminated between equations (A-2)–(A-3) and (A-6)–(A-7), and complex variables can be introduced to yield the following force equation for the projectile:

$$mV \left(\dot{\tilde{\xi}} - i Q \right) = F_y + i F_z - m_c \ell \ddot{E} , \tag{A-10}$$

where

$$\tilde{\xi} = \tilde{\beta} + i\,\tilde{\alpha},$$

$$Q = \dot{\theta} + i\,\dot{\psi},$$

and

$$E = \left(y_c + i\,z_c\right)\ell^{-1}.$$

The external aerodynamic force can be approximated by the linear normal force, and the small component inertia force can be neglected to yield the simple relation

$$m\,V\left(\dot{\tilde{\xi}} - i\,Q\right) = -\,F_N\,\tilde{\xi}, \tag{A-11}$$

where

$$F_N = \left(1/2\right)\rho\,V^2\,S\,C_{N_\alpha}.$$

Equations (A-2)–(A-9) can now be combined to eliminate the transverse forces and moments acting on the internal component. The resulting moment equation for the projectile is

$$I_t\,\dot{Q} - iL_xQ = M_y + i\,M_z - i\,B_\varepsilon\,\ddot{E} - i\,I_{tc}\,\ddot{\Gamma} - p_c\,I_{xc}\,\dot{\Gamma}, \tag{A-12}$$

where

$$L_x = p_b\,I_{xb} + P_c\,I_{xc},$$

$$B_\varepsilon = m_c\,x_c\,\ell,$$

and

$$\Gamma = \gamma_y + i\gamma_z.$$

The external aerodynamic moment can be replaced by the usual linear moment expansion and Q can be eliminated by the use of equation (A-11) to yield the following form of the projectile moment equation:

$$I_t \ddot{\tilde{\xi}} - \left(A_q + i L_x\right) \dot{\tilde{\xi}} - \left(A_\alpha + i\, A_{p\alpha}\right) \tilde{\xi} - B_\varepsilon \ddot{E} - I_{tc} \ddot{\Gamma} + i\, p_c\, I_{xc}\, \dot{\Gamma} = 0, \qquad \text{(A-13)}$$

where

$$A_q = (1/2)\, \rho\, S\, \ell^2\, V \left[C_{M_q} + C_{M\dot{\alpha}} - k_t^2\, C_{N_\alpha}\right],$$

$$A_\alpha = (1/2)\, \rho\, S\, \ell\, V^2\, C_{M_\alpha},$$

and

$$A_{p\alpha} = (1/2)\, \rho\, S\, \ell^2\, V\, p_b \left[C_{M_{p\alpha}} + k_a^2\, C_{N_\alpha}\right].$$

The complex equations for the motion of the internal component can be written from equations (A-6)–(A-9) as follows:

$$m_c \left[\ell\, \ddot{E} + V \left(\dot{\tilde{\xi}} - i\, Q\right) - i\, x_c\, \dot{Q}\right] = F_{yc} + i\, F_{zc} \qquad \text{(A-14)}$$

and

$$I_{tc} \left(\ddot{\Gamma} - i\, \dot{Q}\right) - i\, p_c I_{xc} \left(\dot{\Gamma} - i\, Q\right) = -\, i \left(M_{yc} + i\, M_{zc}\right). \qquad \text{(A-15)}$$

The motion of the internal component is determined by equations (A-11), (A-14), and (A-15) when the component forces and moments are specified and the projectile motion is known.

APPENDIX B:

BEAM INFLUENCE COEFFICIENTS

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If a mass is mounted on an elastic beam and the beam is deformed, the beam exerts a force and moment on the mass. The components of this force and moment appear in equations (A-14) and (A-15) and it is necessary to express these quantities as functions of the displacement (ε_y, ε_z) and rotation (γ_y, γ_z) of the mass. For an elastic beam, these are linear relations:

$$F_{y_c} + i F_{zc} = b_{11}\, \ell E + b_{12}\, \Gamma \qquad \text{(B-1)}$$

and

$$M_{yc} + i M_{zc} = i\,(b_{21}\, \ell E + b_{22}\, \Gamma), \qquad \text{(B-2)}$$

where $b_{12} = b_{21}$.

For simple beam theory, it is easy to express the displacement and rotation as functions of applied force and moment (F_a, M_a). In the xy plane, this yields the following linear relations in terms of the beam influence coefficients:

$$\ell \varepsilon_y = a_{11}\, F_a + a_{12}\, M_a \qquad \text{(B-3)}$$

and

$$\gamma_y = a_{21} F_a + a_{22}\, M_a. \qquad \text{(B-4)}$$

F_a and M_a are the negatives of the corresponding force and moment on the internal component (F_{y_c}, M_{z_c}). Thus, by equations (B-1) to (B-4), the b_{ij}'s are related to the a_{ij}'s as follows:

$$b_{11} = \frac{-a_{22}}{D}, \qquad \text{(B-5)}$$

$$b_{21} = b_{12} = \frac{a_{12}}{D}, \qquad \text{(B-6)}$$

$$b_{22} = \frac{-a_{11}}{D}, \qquad \text{(B-7)}$$

and

$$D = a_{22}\, a_{11} - (a_{12})^2. \qquad \text{(B-8)}$$

We will assume the beam has a length a + b; the internal component and its associated applied force and moment are located a distance a from the forward end of the beam and a distance b from its rear end.

The influence coefficients can be computed for various beams by means of the well-known elastic beam equation:

$$\hat{E}\, I \, \frac{d^2 y}{d x^2} = -M_z(x), \tag{B-9}$$

where

$$\begin{aligned} M_z &= -x F_{yo} + M_{zo} && o \le x < a \\ &= -x F_{yo} + M_{zo} - (x-a)\, F_a + M_a && a \le x \le a+b \\ &= -M_{z1} && x = a+b \end{aligned} \tag{B-10}$$

and

$$F_{yo} + F_{y1} = -F_a, \tag{B-11}$$

where $\hat{E}$ is Young's modulus, I is the area moment of inertia, (M_{zo}, M_{zl}) and (F_{yo}, F_{yl}) are the moments and forces exerted on the ends of the beam.

Three possible constraints are considered at each end of the beam. At the front end, x = o, these are:

(a) free $\quad F_{yo} = 0 \;; \quad M_{zo} = 0$

(b) pinned $\quad y_o = 0$

and

(c) fixed $\quad y_o = 0 \;; \quad \dfrac{dy(0)}{dx} = 0.$

Similar conditions can be assigned to the rear end, $x = a + b$. The values of the influence coefficients are given in Table 1 for six combinations of end conditions: fixed-fixed, fixed-pinned, pinned-fixed, fixed-free, free-fixed, and pinned-pinned. The construction of this table can be illustrated by considering the rearward-facing cantilever case. For this case, $F_{yo} = -F_a$, $M_{zo} = -M_a - aF_a$ (i.e., the beam is fixed-free). Equation (B-9) can be integrated twice to yield

$$\hat{E}I \frac{dy}{dx} = -\frac{x(x-2a)}{2} F_a + xM_a \qquad o \le x < a$$

$$= \frac{a^2}{2} F_a + aM_a \qquad a \le x \le a + b \tag{B-12}$$

and

$$\hat{E}Iy = \frac{-x^2(x-3a)}{6} F_a + \frac{x^2}{2} M_a \qquad o \le x < a$$

$$= \frac{a^2(3x-a)}{6} F_a + \frac{a(2x-a)}{2} M_a \qquad a \le x \le a + b. \tag{B-13}$$

At $x = a$, equations (B-12) and (B-13) become

$$\hat{E}Iy(a) = \hat{E}I\ell\varepsilon_y = \frac{a^3}{3} F_a + \frac{a^2}{2} M_a \tag{B-14}$$

and

$$\hat{E}I\frac{dy(a)}{dx} = \hat{E}I\gamma_y = \frac{a^2}{2} F_a + aM_a . \tag{B-15}$$

The above coefficients of F_a and M_a are precisely those given by the fixed-free entries in Table 1. The other coefficients can be computed in a similar manner.

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LIST OF SYMBOLS

$A_{p\alpha}$	$(1/2)\rho S\ell^2 V p_b [C_{M_{p\alpha}} + k_a^2 C_{N_\alpha}]$
A_q	$(1/2)\rho S\ell^2 V [C_{M_q} + C_{M_{\dot\alpha}} - k_t^2 C_{N_\alpha}]$
A_α	$(1/2)\rho S\ell V^2 C_{M_\alpha}$
a	distance from the forward end of the beam to the internal component
a_{ij}	beam influence coefficients; Table 1
B_γ	$I_{xc}\, p_c - I_{tc}\, \dot\phi_1$
B_ε	$m_c\, x_c\, \ell$
b	distance from the rear end of the beam to the internal component
b_{ij}	beam coefficients derived from the a_{ij}'s.
$C_{M_{p_\alpha}}$	Magnus moment coefficient
$C_{M_q} + C_{M_{\dot\alpha}}$	sum of the damping moment coefficients
C_{M_α}	static moment coefficient
C_{N_α}	normal force coefficient
CM	center of mass
d_γ	nondimensional beam damping orientation coefficient, equation (19)
d_ε	nondimensional beam damping displacement coefficient, equation (18)
E	$(y_c + iz_c)/\ell$
$\hat{E}$	Young's modulus
F_{yc}, F_{zc}	lateral components of the force exerted by the elastic beam on the component

I_x, I_t	axial and transverse moments of inertia of the total projectile relative to its CM
I_{xb}, I_{tb}	axial and transverse moments of inertia of the body relative to its CM
I_{xc}, I_{tc}	axial and transverse moments of inertia of the component relative to its CM
K_j	$\lvert\tilde{\xi}\rvert$
k_a	$\sqrt{I_x/m\ell^2}$, the axial radius of gyration
k_t	$\sqrt{I_t/m\ell^2}$, the transverse radius of gyration
L_x	$p_b\, I_{xb} + p_c\, I_{xc}$
ℓ	reference length
M_{yc}, M_{zc}	lateral components of the moment exerted by the elastic beam on the component
m	$m_b + m_c$
m_b	mass of the body
m_c	mass of the component
p	projectile spin rate
p_b	body spin rate
p_c	component spin rate
Q	$\dot{\theta} + i\dot{\psi}$
S	$\pi\ell^2/4$, reference area
s_g	gyroscopic stability factor, equation (7)
t	time
V	projectile velocity
x	axial distance, measured positive rearward, where $x = 0$ at the forward end of the beam, $x = a$ at the component, and $x = a + b$ at the rear end of the beam
x_b	distance from the CM of the (body + component) to the CM of the body

x_c	distance from the CM of the (body + component) to the CM of the component, where $m_b x_b + m_c x_c = 0$
y_c, z_c	transverse displacements of the component CM relative to the body's axis of symmetry
$\tilde{\alpha}, \tilde{\beta}$	the projectile's angles of attack and sideslip in a nonrolling system
Γ	$\gamma_y + i\,\gamma_z$
γ	$\lvert\Gamma\rvert$
γ_y, γ_z	orientation angles defining the component axis of symmetry with respect to the projectile axis of symmetry
ε	$\lvert E\rvert$
$\dot{\theta}$	projectile pitch rate
λ_1	$\dot{K}_1/K_1$, a damping coefficient
$\tilde{\xi}$	$\tilde{\beta} + i\,\tilde{\alpha}$
ρ	air density
ϕ_1	polar angle of $\tilde{\xi}$
$\dot{\phi}_1$	fast coning frequency
ϕ_γ	Γ phase angle, equation (9)
ϕ_ε	E phase angle, equation (10)
$\dot{\psi}$	projectile yaw rate
$\omega_{\dot{\varepsilon}}$	natural frequency of beam
$(\dot{\ })$	d( )/dt

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